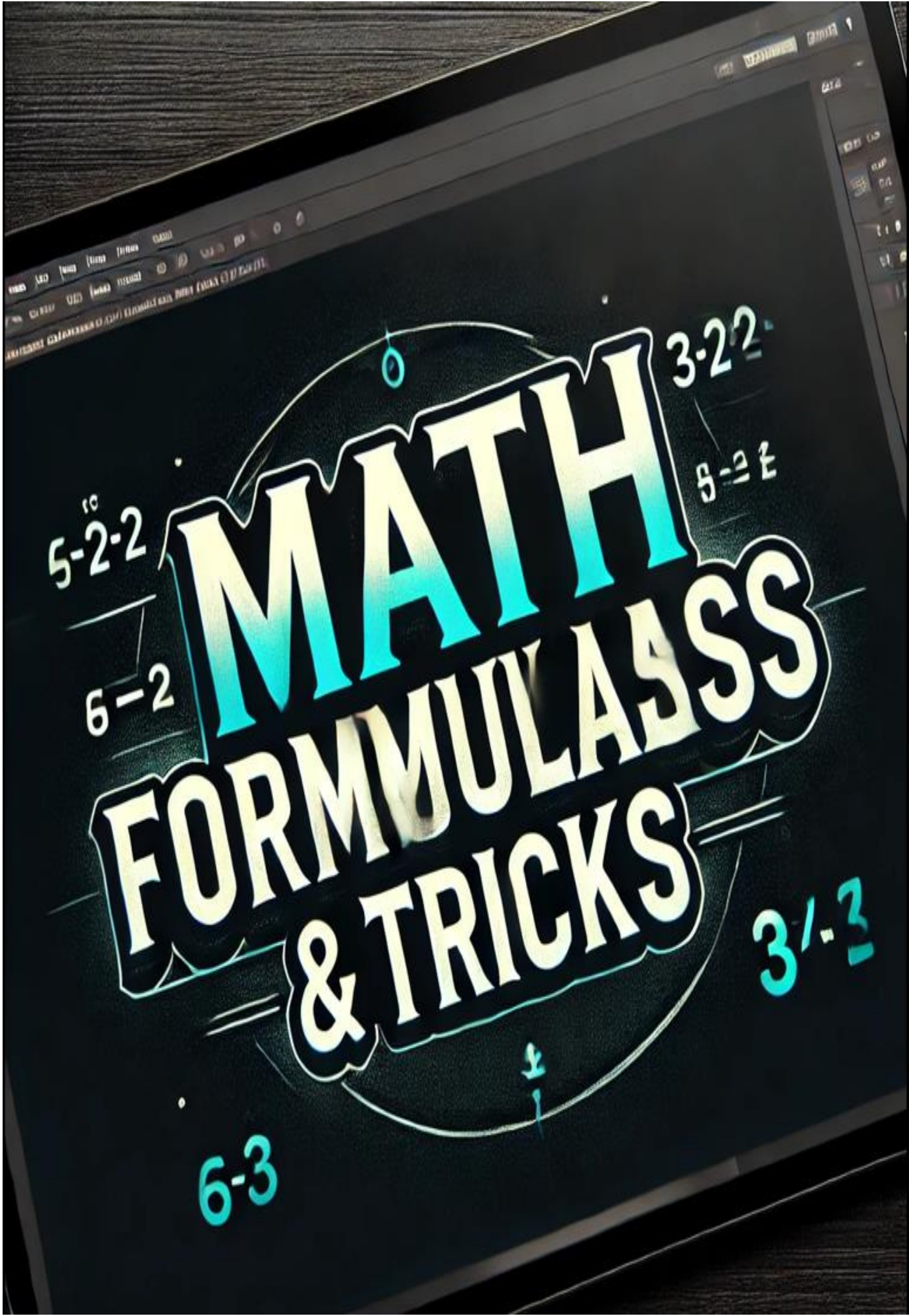
MATH
FORMMULASS
& TRICKS
5-2-2
6-2
6-3

DEDICATION

This book is dedicated to my family for all their support and care and to MR Udom Olusegun.

FOREWORD

There are different keys to opening different doors. Using the wrong key to open a door brings frustration and unnecessary stress to anyone trying to open such a door; however, it is pleasant and pleasurable to use the right keys to any door.

My students dread Mathematics because they are not privy to appropriate formulas and most importantly, tricks and skills needed to ace exams.

It is the passion, zeal, and desire to help students solve these problems that propel Oluwatobiloba Adejumo to write this book – Math Formulas and Tricks.

This book pulls down every mountain students have built around Mathematics and furnishes students with tools, formulas, tricks, and faster ways to solving questions.

With this book, CONFIDENCE IS BUILT, AND SUCCESS IS ACHIEVED.

OLATUBOSUN OLAYEMI

MATH INSTRUCTOR, MANAGEMENT EDUCATION & TRAINING LIMITED.

About The Book

This book is primarily for O'level students containing Math formulas and Math Tricks as is said by the title.

The formulas are the basic formulas you'll need for your exams. Some of these formulas are shorter ways of solving an equation so they help to make your works easier. The book helps to serve as a formula book, so it basically contains only the formulas so you can easily just check for formulas whenever you want to. The next edition, however, will contain examples and exercises to help you better understand the formulas incase you don't already have a book for that.

The tricks are easier methods for solving questions to help reduce the time spent in solving those questions. The tricks can be used in objective exams to help you finish faster to have time to cross-check, but note that these tricks cannot be used in theory exams.

How To Read This Book

For a complete understanding, this book is to be read from beginning to end. Each chapter has a whole different topic so conveys different ideas. And all chapters in the Math Tricks Section come with exercises that should be able to be done within 5 seconds if the tricks are fully understood.

TABLE OF CONTENTS

Basic Facts

This section just comprises various basic things in O' level Mathematics students should know by heart. If you go for competitions, this will be very useful to you, especially if those competitions are quiz competitions.

At the end of this chapter, you should be able to state all these facts without looking at them.

PRIME NUMBERS

Range	Prime Numbers	Number of Prime Numbers	Sum of Prime Numbers
1 – 10	2, 3, 5, 7	4	17
11 – 20	11, 13, 17, 19	4	60
21 – 30	23, 29	2	52
31 – 40	31, 37	2	68
41 – 50	41, 43, 47	3	131
51 – 60	53, 59	2	112
61 – 70	61, 67	2	128
71 – 80	71, 73, 79	3	223
81 – 90	83, 89	2	172
91 – 100	97	1	97

From the table above, you will find out that there are ‘25’ prime numbers from one to hundred.

You will also find out that the sum of prime numbers from 1 – 100 is 1060.

The largest prime number from 1 – 100 is ‘97’.

DICE PROBABILITY

For 1 Dice:

Probability of getting any number: $\frac{1}{6}$

Probability of getting an odd number: $\frac{1}{2}$

Probability of getting a prime number: $\frac{1}{2}$

Probability of getting an even number: $\frac{1}{2}$

Probability of getting a number divisible by 3: $\frac{1}{3}$

For 2 Dice:

Probability of getting the following sum:

Sum of dice	Number of Occurrence	Probability
2	1	$\frac{1}{36}$
3	2	$\frac{2}{36}$ or $\frac{1}{18}$
4	3	$\frac{3}{36}$ or $\frac{1}{12}$
5	4	$\frac{4}{36}$ or $\frac{1}{9}$
6	5	$\frac{5}{36}$
7	6	$\frac{6}{36}$ or $\frac{1}{6}$
8	5	$\frac{5}{36}$
9	4	$\frac{4}{36}$ or $\frac{1}{9}$
10	3	$\frac{3}{36}$ or $\frac{1}{12}$
11	2	$\frac{2}{36}$ or $\frac{1}{18}$
12	1	$\frac{1}{36}$

The total sample space is 36

NUMBER SYSTEM

Binary Number: Base 2

Number	Binary	Number	Binary
1	1	11	1011
2	10	12	1100
3	11	13	1101
4	100	14	1110
5	101	15	1111
6	110	16	10000
7	111	17	10001
8	1000	18	10010
9	1001	19	10011
10	1010	20	10100

Base 3: Ternary

Base 4: Quaternary

Base 5: Quinary

Base 6: Senary

Base 7: Septenary

Base 8: Octal

Base 9: Nenary

Base 10: Decimal

Base 11: Undecimal

Base 12: Duodecimal

Base 60: Vigesimal

Time

	Hours	Minutes	Seconds
Day	24	1 440	86 400
Week	168	10 080	604 800
Month (31 days)	744	44 640	2 678 400
Year	8 760	525 600	31 536 000
Leap year	8 784	527 040	31 622 400

Number of Days in:

Each Third	
1st Third	120 days
2nd Third	123 days
3rd Third	122 days

Each Quarter	
1st Quarter	90 days
2nd Quarter	91 days
3rd Quarter	92 days
4th Quarter	92 days

Each Half	
1st Half	181 days
2n Half	184 days

Measurement

Distance: Meter

Area: $metre^2$

Volume: $metre^3$

Capacity: Litre

Main Conversion

Main unit: meter, litre, etc.

10 milli – 1 centi

10 centi – 1 deci

10 deci – 1 main unit

10 main unit – 1 deca

10 deca – 1 hecto

10 hecto – 1 kilo

Other Conversions

Distance:

1 yard = 3 feet

1 foot = 12 inches

1 mile = 1.6 kilometer

1 nautical mile = 1.85 kilometer

Area:

$1m^2 = 10.76ft^2$

$1ft^2 = 929cm^2$

1 $mile^2$ = 640 acre

1 acre = 43 560ft^2

Pythagorean Triples

3, 4, 5	9, 12, 15	14, 48, 50
5, 12, 13	9, 40, 41	15, 36, 39
6, 8, 10	10, 24, 26	18, 24, 30
7, 24, 25	12, 16, 20	27, 36, 45
8, 15, 17	12, 35, 37	

BASIC FORMULAS

This section shows formulas for the major topics in O' level Mathematics and also comprises shorter formula methods to solving some questions.

At the end of this chapter, you should be able to state all these formulas without looking at them. You should also be able to know what each variable in the formulas mean and when to use those formulas.

Plane Shapes

Shape	Area	Perimeter
Triangle	$\frac{1}{2} * base * height$ or $\frac{1}{2} absin\theta$ θ is the angle between the sides, a and b	a + b + c
Square	Length^2	4 * length
Rectangle	Length * breadth	2(length + breadth)
Parallelogram	Base * perpendicular height	2(a + b), a, b are the different side lengths
Rhombus	0.5 * diagonal_1 * diagonal_2	4 * length
Trapezium	$\frac{1}{2}\ (a+b)h$, h is the height a, b are the parallel sides' lengths	a + b + c + d a, b, c, d are the 4 side lengths
Circle	πr^2, r is the radius	$2\pi r$, r is the radius

Special notes about triangles:

- Area of an equilateral triangle with a given side: $\frac{a^2\sqrt{3}}{4}$

  where a is the length of the side of the triangle

- Area of an equilateral triangle with a given altitude: $\frac{x^2\sqrt{3}}{3}$

  where x is the altitude

- To find altitude of an equilateral triangle with a given side: $\frac{a\sqrt{3}}{2}$

  where a is the given side of the triangle

- Heron's Formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$

  where A is the area of the triangle

  a, b, c are the triangle's side lengths

  $s = \frac{a+b+c}{2}$

Extras about circles:

- Area of sector: $\left(\frac{\theta}{360}\pi r^2\right)$
- Area of segment: Area of sector – Area of triangle

$$\left(\frac{\theta}{360}\pi r^2\right) - \left(\frac{1}{2}r^2 sin\theta\right)$$

Circle Ratios:

- Ratio of radius to circumference

  $1: 2\pi$

- Ratio of diameter to circumference

  $1: \pi$

- Ratio of radius to area

  $1: \pi r$

- Ratio of diameter to area

  $2: \pi r$

- Ratio of area to circumference

  $r: 2$

Equation of a circle:

$(x-a)^2 + (y-b)^2 = r^2$ where point (a, b) is the centre of the circle

or

$x^2 + y^2 - 2ax - 2by = r^2 - a^2 - b^2$

Solid Shapes

Volume of Prisms: Base area * height

Volume of Pyramids: $\frac{1}{3} * base\ area * height$

Solid Shapes	Volume	Total Surface Area	Curved Surface Area
Cube	L^3	$6l^2$	-
Cuboid	Lbh	2(lb + bh + lh)	-
Cylinder	$\pi r^2 h$	$2\pi r(r + h)$	$2\pi rh$
Cone	$\frac{1}{3}\pi r^2 h$	$\pi r(r + l)$	πrl
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$	-

Solid Shapes	Vertices	Edges	Faces
Cuboid	8	12	6
Cube	8	12	6
Rectangular Pyramid	5	8	5
Square based Pyramid	5	8	5
Cone	1		2
Cylinder	0		3

Formula for: Number of Prism faces = $n + 2$

Number of Prism edges = $3n$

Number of Pyramid faces = $n + 1$

Number of Pyramid edges = $2n$

Cylinder Ratios:

- Ratio of radius to surface area

  $1: 2\pi(r+h)$

- Ratio of diameter to surface area

  $1: \pi(r+h)$

- Ratio of radius to volume

  $1: \pi rh$

- Ratio of diameter to volume

  $2: \pi rh$

- Ratio of surface area to volume

  $2(r+h): rh$

Cone Ratios:

- Ratio of radius to surface area

  $1: \pi(r+l)$

- Ratio of diameter to surface area

  $2: \pi(r+l)$

- Ratio of radius to volume

  $3: \pi rh$

- Ratio of diameter to volume

  $6: \pi rh$

- Ratio of surface area to volume

  $3(r+l): rh$

Sphere Ratios:

- Ratio of radius to surface area

  $1: 4\pi r$

- Ratio of diameter to surface area

  $1: 2\pi r$

- Ratio of radius to volume

  $3: 4\pi r^2$

- Ratio of diameter to volume

  $3: 2\pi r^2$

- Ratio of surface area to volume

  $3: r$

Formula for converting a sector to a cone: $\frac{\theta}{360} = \frac{r}{l}$

where θ is the angle of the sector

r is the radius of the cone

l is the slant height of the cone

Polygons

Number of sides	Name of polygon	Sum of interior angles	Each interior angle	Each exterior angle
3	Triangle	180^{o}	60^{o}	120^{o}
4	Quadrilateral	360^{o}	90^{o}	90^{o}
5	Pentagon	540^{o}	108^{o}	72^{o}
6	Hexagon	720^{o}	120^{o}	60^{o}
7	Heptagon	900^{o}	$128\frac{4}{7}^{o}$	$51\frac{3}{7}^{o}$
8	Octagon	1080^{o}	135^{o}	45^{o}
9	Nonagon	1260^{o}	140^{o}	40^{o}
10	Decagon	1440^{o}	144^{o}	36^{o}

Note: There is no polygon with 2 sides or 1 side.

- Sum of interior angles of a polygon: $180(n-2)$
- Each interior angle of a regular polygon: $\frac{180(n-2)}{n}$
- Sum of exterior angles of all polygons is 360^{o}.
- Each interior angle + each exterior angle of a regular polygon = 180^{o}.
- Number of triangles in a polygon: $n-2$ where n is the number of sides of the polygon
- Number of diagonals in a polygon: $\frac{n(n-3)}{2}$ where n is the number of sides of the polygon

TRIGONOMETRY

$Sin\ \theta = \frac{Opposite}{Hypotenuse}$

$Cosine\ \theta = \frac{1}{Sin\ \theta}$

$Cos\ \theta = \frac{Adjacent}{Hypotenuse}$

$Sec\ \theta = \frac{1}{Cos\ \theta}$

$Tan\ \theta = \frac{Sin\ \theta}{Cos\ \theta}$

$Tan\ \theta = \frac{Opposite}{Adjacent}$

$Cot\ \theta = \frac{1}{Tan\ \theta}$

Identities: $sin^2 + \ cos^2 = 1$

$$1 + \ tan^2\theta = sec^2\theta$$

$$1 + cot^2\theta = cosec^2\theta$$

$$\sin\theta = \cos\,(90 - \ \theta)$$

$$\sin(a\ \pm b) = sinacosb\ \pm cosasinb$$

$$\cos(a\ \pm b) = \ cosacosb\ \mp sinasinb$$

$$\tan(a\ \pm b) = \frac{\tan a \pm \tan b}{1 \mp (\tan a)(\tan b)}$$

Differentiation of trig:

	Derivative
Sin θ	Cos θ
Cos θ	-Sin θ
Tan θ	$Sec^2\ \theta$
Cosec θ	-Cot θCosec θ
Sec θ	sec θtan θ
Cot θ	$-cosec^2\ \theta$

Special angles:

Angle	Sin	Cos	Tan
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}\ or\ \frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}\ or\ \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}\ or\ \frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	Undefined

If a + b = 90^0, a and b are complementary angles.

If a + b = 180^0, a and b are supplementary angles.

Lines

Standard form of equation of a line: $y = mx + c$

where y in the dependent variable

x is the independent variable

m is the gradient of the line

c is the y-intercept of the line

Gradient of a Line: $\frac{y_2 - y_1}{x_2 - x_1}$

Angle between two lines: $\text{Tan}^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

where m_1 and m_2 are the gradients of the lines

For parallel lines, $m_1 = m_2$ where m_1 and m_2 are the gradients of the lines

For perpendicular lines, $m_1 = \frac{-1}{m_2}$ where m_1 and m_2 are the gradients of the lines

Notes about lines:

- The gradient of a line parallel to the y-axis is undefined.
- The gradient of a line parallel to the x-axis is 0.
- The gradient of the turning point of any curve is 0.
- The graph of $y = f(x) + a$ is obtained by shifting the graph of $y = f(x)$ a units upwards.
- The graph of $y = f(x) - a$ is obtained by shifting the graph of $y = f(x)$ a units downwards.
- The graph of $y = f(x + a)$ is obtained by shifting the graph of $y = f(x)$ a units to the left.
- The graph of $y = f(x - a)$ is obtained by shifting the graph of $y = f(x)$ a units to the right.
- The graph of $y = -f(x)$ is obtained by reflecting the graph of $y = f(x)$ on the x-axis.
- The graph of $y = f(-x)$ is obtained by reflecting the graph of $y = f(x)$ on the y-axis.

- The graph of $y = |f(x)|$ is obtained by retaining the positive values of $f(x)$ and reflecting the negative values on the x-axis.

Indices & Logarithms

Indices

- $a^x * a^y = a^{x+y}$
- $a^x \div a^y = a^{x-y}$
- $a^x * b^x = (ab)^x$
- $a^{x^y} = a^{xy}$
- $a^{-x} = \frac{1}{a^x}$
- $a^{\frac{x}{y}} = \sqrt[y]{a^x}$
- $\frac{d}{dx} b^x = b^x \ln(b)$

Logarithms

- $\log a + \log b = \log ab$
- $\log a - \log b = log \frac{a}{b}$
- $alog\, b = \log b^a$
- $\log_b a = \frac{\log a}{\log b}$
- $\log_a a^x = x$
- $\log_{b^y} a^x = \frac{x}{y} \log_b a$
- $\frac{d}{dx} \ln(ax^b) = \frac{bax^{b-1}}{ax^b}$

Quadratics

Forms of a quadratic equation:

- Standard form: $ax^2 + bx + c = 0$

  Sum of roots: $\frac{-b}{a}$ Product of roots: $\frac{c}{a}$

  The standard form can also be put: $x^2 - (sum\ of\ roots)x + product\ of\ roots$

- Factored form: $(x - a)(x - b) = 0$ where a and b are the roots of the quadratic equation

- Vertex form: $y = a(x - h)^2 + k$ where point (h, k) is the vertex of the curve

  $h = \frac{-b}{2a}$ and k is the maximum or minimum value of the quadratic equation

Discriminant of a quadratic: $b^2 - 4ac$

Note:

- When the discriminant of a quadratic equation $(b^2 - 4ac)$ is greater than 0, the equation has 2 real solutions.
- When the discriminant of a quadratic equation $(b^2 - 4ac)$ is less than 0, the equation has no real solution.
- When the discriminant of a quadratic equation $(b^2 - 4ac)$ is equal to 0, the equation has 1 real solution, meaning the quadratic is a perfect square.

Statistics

Measures of Central Tendency:

- Mean: the average of all values in a set of data.
- Median: the value in the middle of a set of data wen the set of data is arranged in ascending or descending order.
- Mode: the data that appears/occurs most in a set of data.

Mean: $\frac{\sum fx}{\sum f}$ where f is the frequency of the data

x is the value of the data

$\frac{\sum fd}{\sum f} + a$ where f is the frequency of the data

d is the deviation from the assumed mean

a is the assumed mean

Median Position: $\frac{N+1}{2}$ where N is the total frequency of the values in the data set

Mean Deviation: $\frac{\sum f|d|}{\sum f}$ where f is the frequency of the data

d is the deviation of a data from the mean of the data set

Note: d = data - mean

Measures of Dispersion:

- Range: This is the difference between the largest and smallest data in a data set.
- Interquartile Range: This is the difference between the upper quartile and the lower quartile.
- Variance
- Standard deviation

Range: $a - b$ where a is the largest data in the data set

b is the smallest data in the data set

Interquartile Range: a – b where a is the upper quartile

b is the lower quartile

Upper quartile Position: $\frac{3(N+1)}{4}$

Lower quartile Position: $\frac{N+1}{4}$

Variance: $\frac{\sum fd^2}{\sum f}$ where f is the frequency of the data

d is the deviation of a data from the mean of the data set

$\frac{\sum fd^2}{\sum f} - (\frac{\sum fd}{\sum f})^2$ where f is the frequency of the data

d is the deviation of data from an assumed mean

Standard Deviation: $\sqrt{variance}$ = $\sqrt{\frac{\sum fd^2}{\sum f}}$

where f is the frequency of the data

d is the deviation of a data from the mean of the data set

$= \sqrt{\frac{\sum fd^2}{\sum f} - (\frac{\sum fd}{\sum f})^2}$

where f is the frequency of the data

d is the deviation of data from an assumed mean

Combinatorics

Factorial: $n! = n(n-1)(n-2)(n-3)\dots(1)$

Note: $0! = 1$

Permutation: This is used to know the number of ways a set of stuff can be arranged.

$$\frac{n!}{(n-r)!}$$

Combination: This is used to know the number of ways a set of stuff can be chosen.

$$\frac{n!}{(n-r)!\,r!}$$

Extras:

- Number of ways of arranging stuff in a circular form: $(n-1)!$
- Number of ways of arranging stuff in a circle that can be flipped (e.g a ring): $\frac{(n-1)!}{2}$

Differentiation & Integration

This section might be hard to understand with just formulas, so I am going to explain some of it.

Differentiation

The derivative of a function, $f(x)$, is given by $f'(x)$.

- **Constant Rule:** $\frac{d}{dx}(c) = 0$

  Example: $\frac{d}{dx}(5) = 0$

- **Constant Multiple Rule:** $\frac{d}{dx}[cf(x)] = cf'(x)$

  Example: $\frac{d}{dx}[4f(x)] = 4f'(x)$

- **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$

  Example: $\frac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$

- **Sum Rule:** $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

- **Difference Rule:** $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

- **Chain Rule:** $\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$

  Example: $\frac{d}{dx}\sin 5x = (\cos 5x)(5) = 5\cos 5x$

- **Product Rule:** $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$

  Example:

  $\frac{d}{dx}[(4x^2)(5x^3 + 1)] = (4x^2)(15x^2) + (8x)(5x^3 + 1) = 60x^4 + 40x^4 + 8x = 100x^4 + 8x$

  Using normal power rule, $(4x^2)(5x^3 + 1) = 20x^5 + 4x^2$

  $$\therefore \frac{d}{dx}[(4x^2)(5x^3 + 1)] = \frac{d}{dx}(20x^5 + 4x^2) = (5)(20x^{5-1}) + (2)(4x^{2-1})$$
  $$= 100x^4 + 8x$$

- **Quotient Rule:** $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g\prime(x)}{[g(x)]^2}$

  Example: $\frac{d}{dx}\left(\frac{4x}{9x^5}\right) = \frac{4(9x^5) - (4x)(45x^4)}{(9x^5)^2} = \frac{36x^5 - 180x^5}{81x^{10}} = -\frac{144x^5}{81x^{10}} = -\frac{16}{9x^5} = -\frac{16x^{-5}}{9}$

  Using normal power rule, $\frac{4x}{9x^5} = \frac{4}{9x^4} = \frac{4x^{-4}}{9}$

  $$\therefore \frac{d}{dx}\left(\frac{4x}{9x^5}\right) = \frac{d}{dx}\left(\frac{4x^{-4}}{9}\right) = \frac{(-4)(4)x^{-4-1}}{9} = -\frac{16x^{-5}}{9}$$

You will notice that both methods give the same answer.

Integration

- **Constant Multiple Rule:** $\int kf(x)dx = k\int f(x)dx$

  Example: $\int 4x^3 dx = 4\int x^3\, dx$

- **Power Rule:** $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$ for $n \neq -1$

  Example: $\int 6x^5\, dx = \frac{6x^{5+1}}{5+1} + c = x^6 + c$

- **Sum Rule:** $\int[f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$

- **Difference Rule:** $\int[f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$

- **Exponential Rule:** $\int e^{ax}dx = \text{a}e^{ax} + c$

  $$\int a^x dx = \frac{a^x}{\ln a} + c$$

  Example: $\int e^{4x} dx = 4e^{4x} + c$

  $$\int 4^x dx = \frac{4^x}{\ln 4} + c$$

- **Absolute Value Rule:** $\int |x|\, dx = \frac{x|x|}{2} + c$

Commercial Arithmetic

Simple Interest: $\frac{PRT}{100}$ where P is the principal

R is the rate

T is time

Compound Interest: $P\left(1+\frac{r}{100}\right)^{t}-P$ where P is the original value (principal)

r is the rate

t is time

Amount: Principal + Interest

$\therefore$ Compound Amount: $P\left(1+\frac{r}{100}\right)^{t}$ where P is the original value (principal)

r is the rate

t is time

Annuity: $A=P\times\frac{1-(1+r)^{-n}}{r}$ where P is the value of each payment

r is the rate

n is time

Amortization: $A=P\frac{r(1+r)^{n}}{(1+r)^{n}-1}$ where A is the payment per period

P is the principal loan

r is the interest rate

n is the time (number of payments)

Algebra

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^2 = (a-b)^2 + 4ab$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a-b)^2 = (a+b)^2 - 4ab$

$a^2 - b^2 = (a-b)(a+b)$

$a^2 + b^2 = (a+b)^2 - 2ab \quad or \quad (a-b)^2 + 2ab$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2$

$2(a^2 + b^2) = (a+b)^2 + (a-b)^2$

$(a+b)^2 - (a-b)^2 = 4ab$

Note: If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$.

Sequence

Arithmetic Sequence

$T_n = a + (n-1)d$ where T_n is the number in that term

a is the first term of the arithmetic progression

n is the term number

d is the common difference

$S_n = \frac{n}{2}[2a + (n-1)d]$

where S_n is the sum of the arithmetic sequence up to the term n

a is the first term of the arithmetic progression

n is the term number

d is the common difference

$S_n = \frac{n}{2}(a + l)$

where S_n is the sum of the arithmetic sequence up to term n

a is the first term of the arithmetic progression

n is the term number

l is the last term of the arithmetic progression

Geometric Sequence

$T_n = ar^{n-1}$ where T_n is the number in that term

a is the first term of the geometric sequence

n is the term number

r is the common ratio

$S_n = \frac{a(r^n-1)}{(r-1)}$ for $r > 1$

where S_n is the sum of the geometric progression up to that term

a is the first term of the geometric sequence

n is the term number

r is the common ratio

$S_n = \frac{a(1-r^n)}{(1-r)}$ for $r < 1$

where S_n is the sum of the geometric progression up to that term

a is the first term of the geometric sequence

n is the term number

r is the common ratio

$S_\infty = na$ for $r = 1$

where S_n is the sum of the geometric progression up to that term

a is the first term of the geometric sequence

n is the term number

r is the common ratio

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

where S_∞ is the sum of the geometric progression up to infinity

a is the first term of the geometric sequence

r is the common ratio

Note: S_∞ does not exist for $|r| \geq 1$

Temperature

From	To	Formula
Celcius	Fahrenheit	$\left(x \times \frac{9}{5}\right) + 32$
Fahrenheit	Celcius	$(x - 32)\frac{5}{9}$
Celcius	Kevin	$x + 273$
Kevin	Celcius	$x - 273$
Fahrenheit	Kevin	$(x - 32)\frac{5}{9} + 273$

MATH TRICKS

This section comprises math tricks to make solving faster and easier.

At the end of this chapter, you should be able to know when to apply these tricks and how to apply these tricks. You should also be able to use them very fast: you'll be able to do this by constantly practicing them.

ADDITION OF NUMBERS

The basic math trick dealing with addition is this:

Addition of Fractions

Crossing Method

Trick: $\frac{a}{b} + \frac{c}{d} = \frac{a(d)+b(c)}{(b)(d)}$

Example: What is $\frac{1}{2} + \frac{1}{3}$?

$\frac{1}{2} + \frac{1}{3} = \frac{3(1)+2(1)}{(3)(2)} = \frac{3+2}{6} = \frac{5}{6}$

Exercises: Using this math trick, solve the following

1. $\frac{2}{5} + \frac{3}{4}$
2. $\frac{4}{7} + \frac{3}{8}$

Equivalent Fractions

Using this method, you should try to make the denominators of both fractions equal using the LCM (Lowest Common Multiple) of the denominators.

Example: What is $\frac{3}{4} + \frac{5}{8}$?

First, try to make the denominators the same.

4 is a factor of 8, so the LCM of 4 & 8 is 8.

$\frac{3}{4} * \frac{2}{2} = \frac{6}{8}$ Now that both denominators are the same, you can add

$$\frac{6}{8} + \frac{5}{8} = \frac{11}{8}$$

Using the Crossing Method,

$\frac{3}{4} + \frac{5}{8} = \frac{3(8)+4(5)}{(4)(8)} = \frac{24+20}{32} = \frac{44}{32} = \frac{11}{8}$. They both give the same answer.

Exercises: Using this math trick, solve the following

1. $\frac{3}{7} + \frac{4}{9}$
2. $\frac{3}{5} + \frac{2}{3}$

In conclusion, The Crossing Method should be used for fractions whose denominators are so different from each other, while the equivalent fractions method should be used for fractions in which a denominator is a multiple of another.

SUBTRACTION OF NUMBERS

The basic math trick dealing with addition is this:

Subtraction of Fractions

The same method is used for this as was used for the addition of fractions. The only difference is the change in sign.

Crossing Method

Trick: $\frac{a}{b} - \frac{c}{d} = \frac{a(d)-b(c)}{(b)(d)}$

Example: What is $\frac{1}{2} - \frac{1}{3}$?

$$\frac{1}{2} - \frac{1}{3} = \frac{3(1)-2(1)}{(2)(3)} + \frac{3-2}{6} = \frac{1}{6}$$

Exercises: Using the math trick, solve the following

1. $\frac{3}{4} - \frac{2}{5}$
2. $\frac{4}{7} - \frac{3}{8}$

Equivalent Fractions

I will just give an example for this:

What is $\frac{3}{4} - \frac{5}{8}$?

$$\frac{3}{4} - \frac{5}{8} = \frac{3(8)-4(5)}{(4)(8)} = \frac{24-20}{32} = \frac{4}{32} = \frac{1}{8}$$

Exercises: Using the math trick, solve the following

1. $\frac{4}{9} - \frac{3}{7}$
2. $\frac{2}{3} - \frac{3}{5}$

For normal addition or subtraction of numbers, one needs “number sense” and to get this, one just needs to keep on familiarizing oneself with additions without using calculators or writing materials.

MULTIPLICATION OF NUMBERS

Here below is a basic multiplication table:

×	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320

You are to complete the multiplication table for 17 – 20.

Multiplication of numbers by 50

Note: 50 = $\frac{100}{2}$

Trick: Instead of multiplying directly by 50, multiply by $\frac{100}{2}$.

Example: What is 44 * 50?

44 * 50 = 44 * $\frac{100}{2}$ = 22 * 100 = 2200

Example: What is 65 * 50?

65 * 50 = 65 * $\frac{100}{2}$ = 32.5 * 100 = 3250

Exercises:

1. 74 *50
2. 87 * 50

Multiplication of number by 25

Note: 25 = $\frac{100}{4}$

Trick: Instead of multiplying directly by 25, multiply by $\frac{100}{4}$

Example: What is 64 * 25?

64 * 25 = 64 * $\frac{100}{4}$ = 16 * 100 = 1600

Example: What is 47 * 25?

47 * 25 = 47 * $\frac{100}{4}$ = 13.75 * 100 = 1375

Exercises: Using the math trick, solve the following

1. 45 * 25
2. 104 * 25

Multiplication of numbers ending with “1”

Trick: a1 * b1 = (ab)(a+b)(1)……(ab), (a+b), and (1) are the hundreds, tens, and units digits respectively.

Example: What is 21 * 31?

21 * 31 = (2*3)(2+3)(1) = 651

Exercises: Using the math trick, solve the following

1. 41 * 31
2. 61 * 71

Multiplication of numbers whose last digit sums up to 10 and other digits are the same

Trick: ab * ad = (a*(a+1))(b*d)……(a*(a+1)) and (b*d) are the digits of the number.

Example: What is 56*54?

56 * 54 = (5*(5+1))(6*4) = (5*6)(24) = 3024

Example: What is 102 * 108?

102* 108 = (10*(10+1))(2*8) = (10*11)(2*8) = 11016

Exercises: Using the math trick, solve the following

1. 74 * 76
2. 127 * 123

Note: This trick does not work for things like 74 * 86 because apart from the last digits summing up to 10, the other digits have to be the same.

Multiplication of numbers by 9

Note: 9 = 10 – 1

Trick: a * 9 = a(10 – 1) = 10a – a

For this, you might need a little bit of number sense.

Example: What is 88 * 9?

88 * 9 = 88(10 – 1) = 880 – 88 = 792

Example: What is 54 * 9?

54 * 9 = 54(10 – 1) = 540 – 54 = 486

If that trick seems difficult, here is another:

Trick: ab * 9 = (ab-a-1)(10-b)……where (ab-a-1) and (10-b) are the digits of the answer.

Using the examples above,

88 * 9 = (88-8-1)(10-8) = 792

54 * 9 = (54-5-1)(10-4) = 486

Exercises: Using the math tricks, solve the following

1. 65 * 99
2. 121 * 99

Multiplication of numbers by 99

The trick for this is very similar to that of the previous

Note: 99 =100 – 1

Trick: a * 99 = a(100 – 1) = 100a – a

For this, you will also need number sense.

Example: What is 9 * 99?

9 * 99 = 9(100 – 1) = 900 – 9 = 891

Example: What is 23 * 99?

23 * 99 = 23(100 – 1) = 2300 – 23 = 2277

If that trick seems difficult, here is another:

Trick: ab * 99 = (ab-1)(100-ab)…….where (ab-1) and (100-ab) are the digits of the answer.

Using the examples above,

9 * 99 = (9-1)(100-9) = 891

23 * 99 = (23-1)(100-23) = 2277

Exercises: Using the math tricks, solve the following

1. 4 * 99
2. 78 * 99

DIVISION OF NUMBERS:

Trivial divisions you should know by heart:

$\frac{1}{2} = 0.5$

$\frac{1}{4} = 0.25$

$\frac{1}{8} = 0.125$

$\frac{3}{4} = 0.75$

$\frac{3}{8} = 0.375$

$\frac{5}{8} = 0.625$

$\frac{7}{8} = 0.875$

Note: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

Division of Odd Numbers by 2

Trick: $\frac{a}{2} = \frac{a-1}{2} + \frac{1}{2}$

Example: What is $\frac{65}{2}$?

$$\frac{65}{2} = \frac{65-1}{2} + \frac{1}{2} = \frac{64}{2} + \frac{1}{2} = 32 + 0.5 = 32.5$$

Exercises: Using the math trick, solve the following

1. $\frac{177}{2}$
2. $\frac{9999999999999999}{2}$

Division of Numbers by 0.2

Trick: $\frac{a}{0.2} = a * 5$ i.e for every number you divide by 0.2, just multiply it by 5 instead.

Example: What is $\frac{73}{0.2}$?

$$\frac{73}{0.2} = 73 * 5 = 365$$

Exercises: Using the math trick, solve the following

1. $\frac{45}{0.2}$
2. $\frac{199}{0.2}$

Division of Numbers by 0.5

Trick: $\frac{a}{0.5} = a * 2$ i.e for every number divided by 0.5, just multiply by 2 instead

Example: What is $\frac{45}{0.5}$?

$$\frac{45}{0.5} = 45 * 2 = 90$$

Exercises: Using the math trick, solve the following

1. $\frac{78}{0.5}$
2. $\frac{187}{0.5}$

Rules of Divisibility

- **Rule for 'Number 1':** All numbers are divisible by 1.
- **Rule for 'Number 2':** If the last digit of the number is an even number, the number is divisible by 2.

  Example:

  20, 22, 24, 26, and 28 are divisible by 2 because their last digits are even numbers.

  But, 21, 23, 25, 27, and 29 are not divisible by 2 because their last digits are odd numbers.
- **Rule for 'Number 3':** If the sum of the digits of the number is a multiple of 3, the number is divisible by 3.

  Example:

  225 is divisible by 3 to give 75.

  Here's proof: 2 + 2 + 5 = 9 and 9 is divisible by 3.

  But 227 is not divisible by 3 because 2 + 2 + 7 = 11 and 11 is not divisible by 3.
- **Rule or 'Number 4':** If the last two digits form a number divisible by 4, the number is divisible 4.

  Example:

  1024 is divisible by 4 to give 256 because 24 is divisible by.

  But, though, 1026 is divisible by 2, it is not divisible by 4 because 26 is not divisible 4.
- **Rule for 'Number 5':** If the last digit of the number is '0' or '5', the number is divisible by 5.

  Example:

  20, 55, 1005, 400 and 235 are all divisible by 5 because their last digits are either '0' or '5'.

- **Rule for 'Number 6':** If the sum of the digits is divisible by 3 and the last digit is an even number, the number is divisible by 6.

  Example:

  414 is divisible by 6 to give 69 because the sum of the digits (4 + 1 + 4) gives 9 which is divisible by 3 and the last digit, 4, is an even number.

  But, even though 417 is divisible by 3, it is not divisible by 6 because the last digit, 7, is and odd number.

- **Rule for 'Number 7':** If you remove the last digit of a number from a number, double that digit and subtract it from the remaining part of the number and the result is divisible by 7, then that number is divisible by 7.

  Example:

  343 is divisible by 7 to give 49.

  Proof: Remove the last digit which is '3' so you get 34. 3 * 2 = 6. 34 – 6 = 28 which is divisible by 7 so 343 is divisible by 7.

- **Rule for 'Number 8':** If the last 3 digits form a number divisible by 8, the number is divisible by 8.

  Example:

  7448 is divisible by 8 to give 931 because the last three digits form a number, 448, that is divisible by 8.

  Also, 1448, 2448, 3448, 5448, 6448, 8448, and 9448 are all divisible by 8 to give 181, 306, 431, 556, 681, 806, 1056, and 1181 respectively because their last digits are 448 which is divisible by 8.

- **Rule for 'Number 9':** If the sum of the digits is divisible by 9, the number is divisible by 9.

  Example:

  486 is divisible by 9 to give 54 because the sum of the digits (4 + 8 + 6) is 18 which is divisible by 9.

  But, even though 15 is divisible by 3, it is not divisible by 9 because the sum of the digits (1 + 5) is 6 which is not divisible by 9.

- **Rule for ‘Number 10’:** If the last digit is ‘0’, the number is divisible by 10.

  Example:

  20, 30, 40, 50, 60, 70, 80, and 90 are all divisible by 10 because their last digit is ‘0’.

- **Rule for ‘Number 11’:** If the difference between the sum of the digits of a number at odd places and the sum of the digits at even places is ‘0’ or a multiple of 11, then that number is divisible by 11.

  Example:

  121 is divisible by 11 because $(1 + 1) - 2 = 0$

  82720 is divisible by 11 because $(8 + 7 + 0) - (2 + 2) = 11$ which is a multiple of 11.

- **Rule for ‘Number 12’:** If the number is divisible by ‘3’ and ‘4’ i.e the sum of the digits of the number is divisible by 3 and the last two digits of the number form a number divisible by 4, the number is divisible by 12.

  Example:

  156 is divisible by 12 to give 13 because the sum of the digits (1 + 5 + 6) is 12 which is divisible by 3 and the last two digits form 56 which is divisible by 4.

- **Rule for ‘Number 13’:** a number is divisible by 13 when the ones place digit of a number is multiplied by 4 and the product when added to the rest of the number either gives 0 or a multiple of 13.

  Example:

  52 is divisible by 13 to give 4.

  Proof: $5 + (2 * 4) = 13$; since the result is 13 which is a multiple of 13, 52 is divisible by 13.

- **Rule for ‘Number 14’:** An even number divisible by 7 is divisible by 14.

  Example:

  14, 28, 42 are all divisible by 14 because they are even numbers divisible by 7.

- **Rule for 'Number 15':** A number is divisible by 15 when it is divisible by both '5' and '3'.

  Example:

  15, 30, 45 are divisible by 15 because they are divisible by both '5' and '3'.

- **Rule for 'Number 16':** If the last four digits of a number are divisible by 16, then the number is divisible by 16.

  Example:

  658320 is divisible by 16 because 8320 is divisible by 16.

- **Rule for 'Number 17':** If you multiply the last digit of a number by 5 and subtract that from the rest of the number and the result is '0' or a multiple of 17 then that number is divisible by 17.

  Example:

  51 is divisible by 17.

  Proof: $5 - (1 * 5) = 0$; since the result is zero, 51 is divisible by 17.

- **Rule for 'Number 18':** If a number is divisible '2' and '9', then it is divisible by 18.

  Example:

  36, 54, 126 are divisible by '2' and '9' and so are divisible by 18.

- **Rule for 'Number 19':** If you multiply the last digit of a number by 2 and add that to the rest of the number and the result is '0' or a multiple of 19 then that number is divisible by 19.

  Example:

  209 is divisible by 19.

  Proof: $20 + (9 * 2) = 38$; since the result is 38 which is a multiple of 19, 209 is divisible by 19.

- **Rule for 'Number 20':** If the last digit is '0' and the second to the last digit is an even number, the number is divisible by 20.

Note: Any number divided by 0 gives 'undefined' as the answer.

Exercises: Using the rules of divisibility, answer the following.

1. Is 457857 divisible by 3? Give a reason for your answer.

2. Is 4367956 divisible by 4? Give a reason for your answer.
3. Is 457857 divisible by 6? Give a reason for your answer.
4. Is 7791 divisible by 7? Give a reason for your answer.
5. Is 3487560 divisible by 8? Give a reason for your answer.
6. Is 45764 divisible by 9? Give a reason for your answer.
7. Is 123464 divisible by 11? Give a reason for your answer.
8. Is 789342 divisible 12? Give a reason for your answer.
9. Is 1391 divisible by 13? Give a reason for your answer.
10. Is 7791 divisible by 14? Give a reason for your answer.
11. Is 731 divisible by 17? Give a reason for your answer.
12. Is 23348 divisible by 19? Give a reason for your answer.

SQUARING OF NUMBERS

Here is a list of the squares of numbers from 1 to 40:

Number	Square	Number	Square
1	1	21	441
2	4	22	484
3	9	23	529
4	16	24	576
5	25	25	625
6	36	26	676
7	49	27	729
8	64	28	784
9	81	29	841
10	100	30	900
11	121	31	961
12	144	32	1024
13	169	33	1089
14	196	34	1156
15	225	35	1225
16	256	36	1296
17	289	37	1369
18	324	38	1444
19	361	39	1521
20	400	40	1600

Squaring of numbers ending with “5”

This trick is basically that of multiplying numbers ending whose last digits sum up to 10.

As you know, 5 + 5 = 10

Example: What is 55^2?

$55^2 = 55 * 55 = (5*(5+1))(5*5) = (5*6)(25) = 3025$

Example: What is 125^2?

125^2 = 125 * 125 = (12*(!2+1))(5*5) = (12*13)(25) = 15625

Exercise: Using the math trick, solve the following

1. 145^2
2. 205^2

Squaring of numbers slightly less or more than 50

Trick: Find the difference between the number and 50. If the number is bigger than 50, add the difference to 25 (this will be the first digits) and square the difference (this will be the last digits), but if the number is less than 50, subtract the difference from 25 (this will be the first digits) and square the difference (this will be the last digits).

Example: What is 47^2?

47 is less than 50, so we are going to subtract the difference from 25.

50 – 47 = 3

25 – 3 = 22; $3^2 = 09$

47^2 = 2209…..the thing is you should already know that the square of 47 is a 4-diit number.

Example: What is 57^2?

57 is more than 50, so we are going to add the difference to 25.

57 – 50 = 7

25 + 7 = 32; $7^2 = 49$

$57^2 = 3249$

Example: What is 65^2?

65 is more than 50, so we are going to add the difference to 25.

$65 - 50 = 15$

$25 + 15 = 40$; $15^2 = 225$

So, I know you will probably be thinking that the answer is 40225, but 65^2 is a 4-digit number, so you add the first 2 to the zero making it 4225.

You can confirm this using the trick for the squaring of numbers whose last digit is 5.

Exercises: Using the math trick, sole the following

1. 44^2
2. 63^2

Squaring of numbers slightly less or more than 100

Trick: Find the difference between the number and 100. If the number is bigger than 100, add the difference to the number (this will be the first digits) and square the difference (this will be the last digits), but if the number is less than 100, subtract the difference from the number (this will be the first digits) and square the difference (this will be the last digits).

Example: What is 94^2?

94 is less than 100, so we are going to subtract the difference from 100.

$100 - 94 = 6$

$94 - 6 = 88$; $6^2 = 36$

$94^2 = 8836$

I know you may be wondering how you will know the number of digits it has. So here is how: 100^2 is 10000 which is the first 5-digit number so obviously 94^2 can not be 5-digit so logically, it has to be a 4-digit number.

Example: What is 108^2?

108 is more than 100, so we are going to add the difference to 100.

$108 - 100 = 8$

$108 + 8 = 116$; $8^2 = 64$

$108^2 = 11664$

Exercise: Using the math trick, solve the following

1. 106^2
2. 91^2